A table of weights and abscissas (to sixteen significant figures) for the $2n$-point Gaussian integration formula

$$\int_{-\pi}^{\pi} f(x) \sin x \, dx = \sum_{j=1}^{n} [w_j f(x_j) - w_j f(-x_j)], \qquad 0 < x_j < \pi,$$

is given for $n = 1(1)9$. The abscissas x_j and weights w_j are given by

$$x_j = \sqrt{u_j}, \qquad w_j = b_j/x_j, \qquad j = 1, 2, \ldots, n,$$

where u_j and b_j are the abscissas and weights of the n-point Gaussian quadrature formula

$$\int_0^{\pi^2} \frac{1}{2} \sin \sqrt{u}\, g(u)\, du = \sum_{j=1}^{n} b_j g(u_j).$$

The author chose the method of Anderson [1], to compute the u_j and b_j. The calculations were performed on the IBM 1620 of the Computing Center of the University of Leuven.

E. I.

1. D. G. Anderson, "Gaussian quadrature formulae for $\int_0^1 -\ln(x) f(x)\, dx$," *Math. Comp.*, v. 19, 1965, pp. 477–481.

25[2.10, 7].—David M. Bishop, *Evaluation of Certain Integrals of Reduced Modified Bessel Functions of the Second Kind*, ms. of 6 typewritten pages (undated) deposited in the UMT file.

The main table in this manuscript consists of 8S floating-point values of the integral

$$I_n(\mu) = \int_0^{\infty} [x^{\mu} K_{\mu}(x)]^4 x^n \, dx$$

for $n = 0(1)4$ and $\mu = 0.1(0.1)5$. The numerical integration was accomplished by means of Gauss-Laguerre quadrature, using 32 points.

When μ is half an odd integer (and n is a nonnegative integer) the value of $I_n(\mu)$ is expressible as a rational multiple of π^2; accordingly, the author presents in a supplementary table these rational coefficients (with their decimal equivalents to 8S) for $n = 0(1)4$ and $\mu = 0.5(1)2.5$. From a comparison of corresponding entries in the two tables the author concludes that the main table is accurate to at least 7S. (This reviewer noted one instance of accuracy to 6S, corresponding to $n = 0$, $\mu = 1.5$.)

The author states that this integral appears in certain atomic and molecular calculations where the electronic wave function is chosen as a combination of Bessel functions.

J. W. W.

26[2.30, 2.45, 8, 9, 12].—Donald E. Knuth, *The Art of Computer Programming*, Vol. II: *Seminumerical Algorithms*, Addison-Wesley Publishing Co., Reading, Mass., 1969, xi + 624 pp., 25 cm. Price $18.50.

In this impressive sequel to the first volume [1] of his projected seven-volume series on the art of computer programming the author considers those aspects of computer programming most closely related to classical mathematics, numerical analysis, and statistics.

As concisely stated in the preface, the problem discussed here is "to find the best ways to make computers deal with numbers".

The present volume consists of two additional chapters in the complete work; namely, Chapter 3, entitled "Random Numbers", and Chapter 4, entitled "Arithmetic".

The introduction to Chapter 3 contains an enumeration of various areas in which random numbers find useful application; of these, that of primary interest in this book is the testing of the effectiveness of computer algorithms by means of such data. The well-known "middle-square" method of generating "random" numbers is carefully examined, and the conclusion is reached that such numbers should not be generated by a method chosen at random. Superior random-number generators, considered next, include the linear congruential method, the quadratic method of Coveyou, the Fibonacci recurrence, and the combination of such generators.

Statistical tests for randomness are treated at length in Section 3.3 (pp. 34–100). These include general tests (the chi-square test and the Kolmogorov-Smirnov test), empirical tests (frequency, serial, gap, poker, coupon-collector's, permutation, and run tests), and theoretical tests in connection with linear congruential sequences. The important spectral test, which was developed in 1965 by Coveyou and MacPherson, is discussed in considerable detail (pp. 82–100).

Next we find in Section 3.4 a useful summarization of the best known procedures for producing numbers from various standard statistical distributions, followed by a discussion of random sampling and shuffling (with algorithms attributed to Ulam and Durstenfeld).

In Section 3.5 (pp. 127–151), which includes historical and bibliographic information, the author considers the basic question as to what constitutes a random sequence. On p. 142 he formulates a definition of such a sequence, which he believes satisfies all reasonable philosophical requirements.

A useful summary of this chapter presents a procedure for obtaining the "nicest" and "simplest" random-number generator and concludes with a brief discussion of additional references for supplementary reading.

Chapter 4 contains detailed analyses of algorithms for performing arithmetic operations on a variety of quantities such as floating-point numbers, multiple-precision numbers, rational numbers, polynomials, and power series. Related topics include radix conversion, factorization of integers, and evaluation of polynomials.

One of the most entertaining and informative sections in this book, in the opinion of the reviewer, is the first one of this chapter. Positional number systems and their historical development are here treated, with appropriate attention to the influence of electronic digital computers. In particular, unusual number systems such as the factorial system and those involving complex as well as negative bases are also considered.

In Section 4.2 single- and double-precision floating-point calculations are successively treated, with particular attention to the inherent accuracy of such calculations performed on electronic computers. An important section describes the common pitfalls encountered in the preparation of floating-point routines. Illustrative programs are written in MIX language, a description of which appears in Appendix A (pp. 565–595), as reproduced from Volume 1 (pp. 120–152).

The general subject of multiple-precision arithmetic is then considered. Algorithms, accompanied by MIX programs, are stated for the four fundamental arithmetic operations and formulas are derived for the corresponding running time in computer cycles.

An alternative approach to performing arithmetical operations on large integers, namely modular arithmetic, is next described and the application of this method to the exact solution of linear equations with rational coefficients is mentioned.

The challenging question, "How fast can we multiply?" is discussed at great length. Apparently, a faster method than the familiar one of order n^2 for multiplying multiple-precision numbers was not discovered until 1962. The improved method of Toom & Cook, especially adapted for electronic computers, and the modular method of Schönhage are both described in detail, including appropriate algorithms.

In Section 4.4 radix conversion is briefly discussed, including four basic methods and illustrative examples involving single- and multiple-precision conversion, floating-point conversion, and hand calculation.

The next topic of wide importance treated here is that of rational arithmetic in Section 4.5 (pp. 290–360). The fundamental algorithm is that of Euclid for finding the greatest common divisor of two integers. The intimate connection between this algorithm and regular continued fractions leads to a discussion of such details as Lamé's theory and the research of Lévy.

The author devotes nearly 22 pages to the perennial problem of factoring integers. The methods described include division by successive primes, Fermat's method, use of sieves, and Legendre's method. Tests of primality, including the Lucas-Lehmer test, are also described. An up-to-date account of the search for Mersenne primes is included. Number theorists should find the list of special prime numbers on p. 355 particularly interesting and useful.

The following discussion of polynomial arithmetic in Section 4.6 leads to a study of addition chains (pp. 398–422) which are encountered in a search for the most economical way to compute x^n by multiplication. The problem of the efficient evaluation of polynomials leads to discussions of such procedures as Horner's rule, Yate's method, Lagrange's and Newton's interpolation formulas, and Pan's method. Finally, the concept of addition chains is extended to polynomial chains in connection with the minimization of the number of arithmetic operations for evaluating polynomials of various degrees.

The concluding section of this chapter deals with the manipulation of power series. The transformation considered in greatest detail here is that of reversion of series, and three algorithms are presented for that purpose.

The wealth of information in this volume is supplemented by more than 650 exercises of graduated difficulty, ranging from trivial problems to unsolved research questions. As in Volume I, a large part of this book (pp. 452–564) is devoted to the answers to the great majority of these exercises.

Tables of important constants to 40 decimal places and 44 octal places, in addition to the first 25 harmonic, Bernoulli, and Fibonacci numbers, are presented in Appendix B (pp. 596–599). A useful index to notations appears in Appendix C (pp. 600–604).

This exceptionally scholarly, informative book is a worthy companion to the first volume of this series. It should be read by all persons aspiring to become experts in the fields of numerical analysis and computer programming, and it certainly can be studied with profit by students in both pure mathematics and statistics.

J. W. W.

1. DONALD E. KNUTH, *The Art of Computer Programming*, Vol. I: *Fundamental Algorithms*, Addison-Wesley Publishing Co., Reading, Mass., 1968. (See *Math. Comp.*, v. 23, 1969, pp. 447–450, RMT **18.**)

27[3].—GEORGE E. FORSYTHE & CLEVE B. MOLER, *Computer Solution of Linear Algebraic Systems*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1967, xi + 148 pp., 24 cm. Price $6.75.

This is an excellent brief introduction to the subject, written by two experts with considerable experience. The senior author, in fact, is one of the pioneers.

Very little is presupposed on the part of the reader. The necessary theoretical background is developed in an elementary fashion, and detailed algorithms are spelled out and analyzed. For the beginner, and even for those who already have some experience, this book is a must.

A. S. H.

28[3, 10].—MORRIS NEWMAN, *Matrix Representations of Groups*, Applied Mathematics Series No. 60, National Bureau of Standards, Washington, D. C., 1968, 79 pp., 26 cm. Price $0.60.

This monograph develops the theory of representations of groups in terms of finite dimensional matrices over the field of complex numbers, with strong emphasis on the representation theory of finite groups. It avoids algebraic machinery outside of matrix theory as far as possible, trying successfully to give proofs which are both elementary and simple. Appendices deal with the elements of the theory of algebraic numbers (needed, e.g., for proving the solvability of groups whose order is divisible by not more than two distinct prime numbers) and specifically with the roots of unity. An interesting proof of the irreducibility of the cyclotomic polynomials is included.

Except for the monograph by Martin Burrow [*Representation Theory of Finite Groups*, 185 pp., Academic Press, New York, 1965] (which goes farther and develops and uses more advanced algebraic tools), there seems to exist no book of comparable size in English which gives the same amount of information; the book by Curtis and Reiner [*Representation Theory of Finite Groups and Associative Algebras*, 685 pp., Interscience, New York, 1962] is much larger, and the book by Marshall Hall [*The Theory of Groups*, 434 pp., Macmillan, New York, 1959], which contains representation theory as a chapter, covers many other parts of group theory as well.

There is no doubt that the present monograph will be useful for many purposes and to many readers. In particular, the explicit construction of a full set of irreducible representations for some finite groups may be welcome to many users.

The reviewer found the book very clear locally, but less so globally. It follows, of course, from the proved results, that all finite dimensional representations of a finite group are equivalent to a matrix representation which is composed in an obvious manner of a finite number of irreducible representations, and that a knowledge of